Chapter

Topology Optimization Applications on Engineering Structures

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Abstract

Over the years, several optimization techniques were widely used to find the optimum shape and size of engineering structures (trusses, frames, etc.) under different constraints (stress, displacement, buckling instability, kinematic stability, and natural frequency). But, most of them require continuous data set where, on the other hand, topology optimization (TO) can handle also discrete ones. Topology optimization has also allowed radical changes in geometry which concludes better designs. So, many researchers have studied on topology optimization by developing/using different methodologies. This study aims to classify these studies considering used methods and present new emerging application areas. It is believed that researchers will easily find the related studies with their work.

Keywords: topology optimization, finite element method

1. Introduction

Topology optimization (TO) is carried out to obtain an optimal structural layout [1]. It is one of the branches of optimization methods differing from size and shape optimization. As expected, as a type of optimization method, it has constant parameters, like applied loads, material type, etc., objective function and constraints which change for every problem, and lastly variable which are the parameters of the material layout. In shape optimization, it aimed to find the position of the member of the structure, while in size optimization, only finding the size of the members is enough. In both cases, there will be no change in the number of members. On the other hand, in topology optimization some part or member of the structure will be deleted and a new layout will be prepared [2]. It is generally preferred to use finite element method (FEM) as meshing eases to find the places to be deleted. But as an optimization algorithm, several kinds are used including both gradient-based such as optimality criteria methods and non-gradient-based algorithms such as genetic algorithm [3].

The topology optimization of structures has proven to be a valuable tool for the identification of the best concepts in early phases of the design process. It is widely used in lightweight design of structures in automotive and aerospace industry, as well as in civil engineering, material science, and biomechanics [1, 4, 5].

This chapter will give brief introduction on topology optimization and later give related studies under several classifications. There are several well-prepared and intensely examined review studies in literature, but some of them are on specific application area (vibration problems [6], continuum structures [7]) or are on a specific methodology (evolutionary algorithms [8, 9], level-set methods [10]), or recent studies are not included [11, 12]. This study mostly aims to present recent studies while giving brief description on previous ones.

2. Topology optimization

During the twentieth century, architects and engineers have used innovative and novel methods to develop optimum forms of structures and sculptures. While the techniques employed by these innovators generated efficient and aesthetic forms, they shared a common limitation: reaching optimum structure. Although the purpose of applying topology optimization has never been a standard procedure, developments in finding optimum structure form let the researchers and designers be free to constructing better designs [13, 14].

Topology optimization offers conceptual design for lighter and stiffer structures. It helps to reach to efficient and aesthetic designs within a small time interval (**Figure 1**). The benefits are:

- Building weight-saving and complete designs.
- Decrease needed time to present and test product.
- By the help of FEM software, you are able to check your design from the perspective of:
    - Determining feasible design range.
    - Accurate checking for different loads and conditions.
    - Considering design and manufacturing constraints [15].

By the time, TO has shown its power and efficiency in the design of structures by the increase in advances on computational speed and power. Changes in computer hardware and software technology have also changed the approach to topology formation of structures. Nowadays, you could use a drawing software in forming different topologies as if it is a standard task, and so, you are able to alter

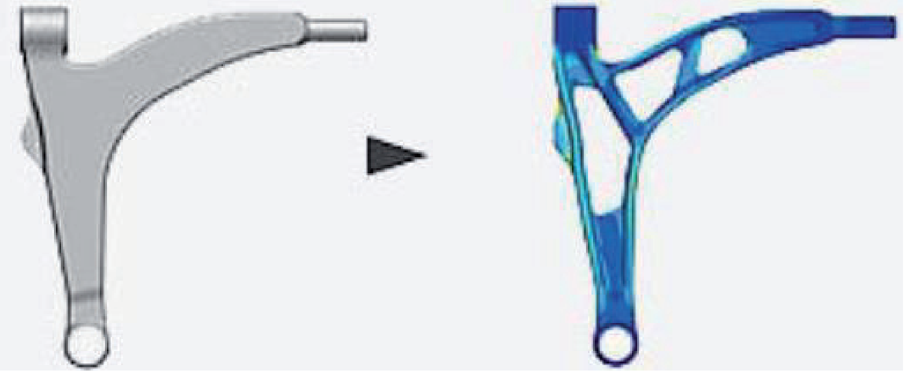

Figure 1.
Optimized unit by using topology optimization approach (Dassault) [15].

old designs and produce new alternative designs in virtual environment. Designers and engineers are pleased to have such a powerful tool in their work [16].

2.1 General form

A topology optimization problem can be written in the general form of an optimization problem as [3, 17]:

$$\begin{aligned} &minimize_{\rho}\, F = F(u(\rho), \rho) = \int_{\Omega} f(u(\rho), \rho) dV \\ &subject\ to\ G_0(\rho) = \int_{\Omega} \rho dV - V_0 \\ &G_j(u(\rho), \rho) \le 0\ with\ j = 1, \ldots, m \end{aligned} \tag{1}$$

The problem statement includes the following:

- An objective function $\boldsymbol{F}(\boldsymbol{u}(\boldsymbol{\rho}), \boldsymbol{\rho})$. Even though each problem could have different objective functions, generally the most used one is minimizing compliance, or in another word, maximizing the stiffness of the structure.

- Main design variable: material distribution. Here material density at each point of the members $\boldsymbol{\rho}(\boldsymbol{u})$ could be this variable. 1 represents the places where density is described, and 0 is for the places where the material is deleted or there is none. On the other hand, $\boldsymbol{u}$ defines if the state is linear or nonlinear [11].

- The design space $\boldsymbol{u}(\boldsymbol{\rho})$. This points out how much volume exists in design. There are many design factors such as manufacturing and handling that should be taken into account in determination of this value. Once this value is determined, then no need to change these places in the optimization stage.

- m constraints is a characteristic that the solution must satisfy $\boldsymbol{G\,j}(\boldsymbol{u}(\boldsymbol{\rho}), \boldsymbol{\rho}) \le \boldsymbol{0}$. The examples are the maximum amount of material to be distributed (volume constraint) or maximum stress values.

- Evaluating $\boldsymbol{u}(\boldsymbol{\rho})$ often includes solving a differential equation. This is most commonly done using the finite element method since these equations do not have a known analytical solution [3].

2.2 Structural topology optimization

The topology of a structure is defined as a spatial arrangement of structural members and joints or internal boundaries. For both discrete and continuum structures, topology optimization helps to arrange association form of members as can be realized in **Figure 2** [18].

The conceptual process is shown in **Figure 3**.

Structural optimization is concerned with maximizing the utility of a fixed quantity of resources to fulfill a given objective. In structural optimization the best "structural" design is selected regarding three categories: size optimization, shape optimization, and topology optimization [19]. The application of topology optimization to structures to reveal the best position and size of the parts in a continuum is the most favorite one. Michell presented the first solutions as seen in **Figure 4**. Today much more advanced techniques are used, and by the help of finite element method, it could be applied to complex problems. Weight savings are managed by engineers in several structures as a consequence of utilization of these

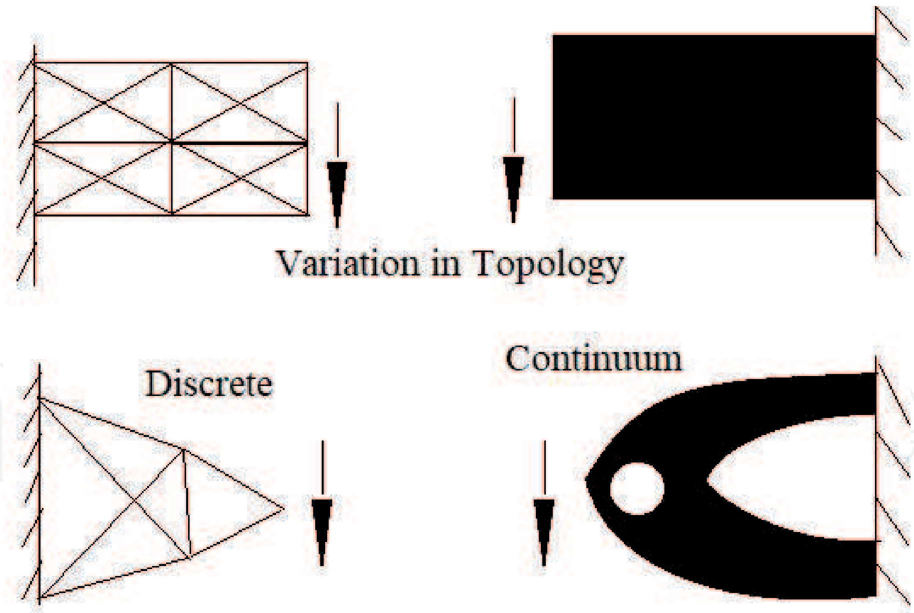

Figure 2.
Variation of topology [18].

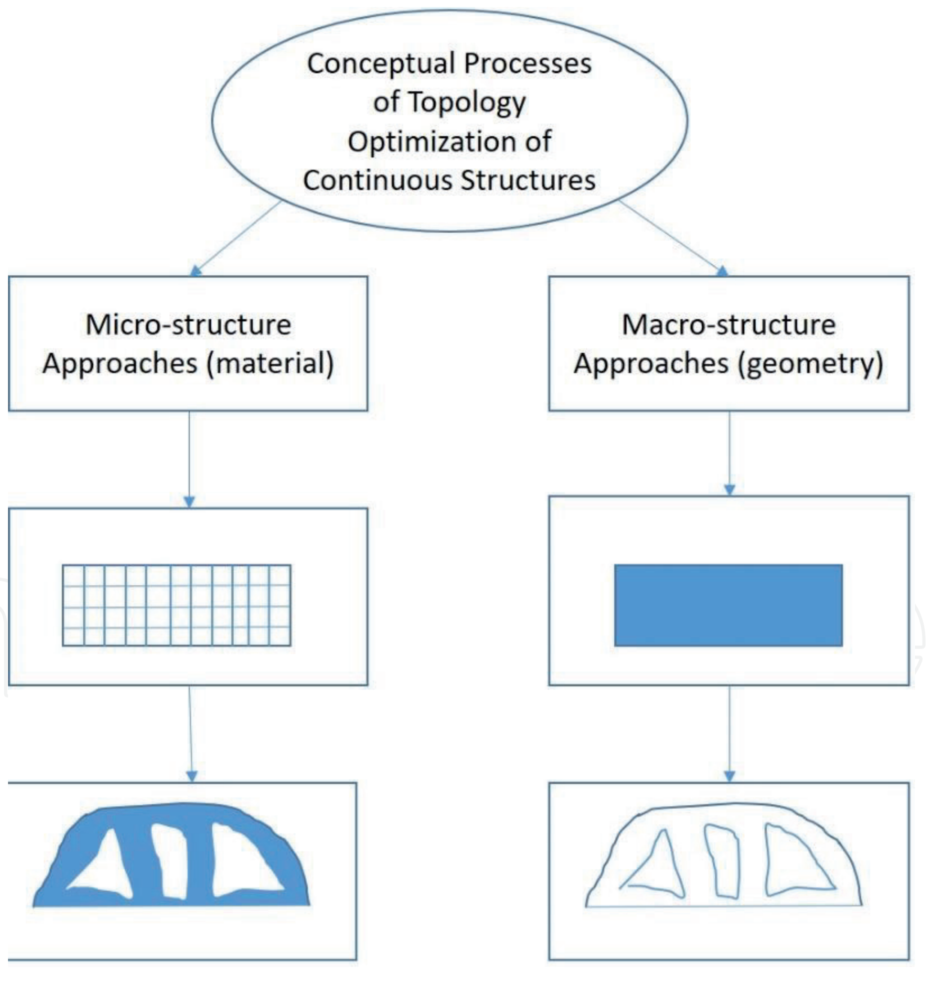

Figure 3.
Conceptual process [18].

methods. There are many examples in literature on the application of these methods [13, 20, 21]. Today, many commercial finite element software has an optimization module (Altair OptiStruct, Simulia Tosca, OPTISHAPE-TS, etc.) to obtain lighter

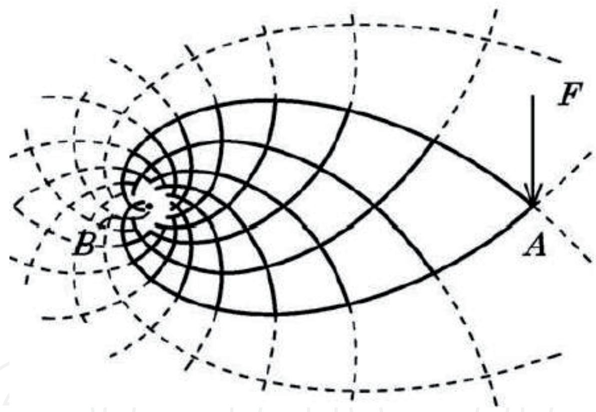

Figure 4.
One of the first proposed solutions to a structural topology optimization problem [13].

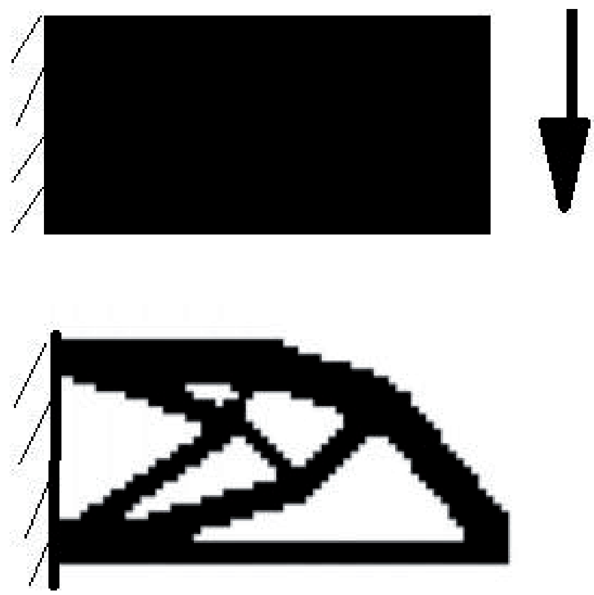

Figure 5.
Initial and optimized unit structure of a short cantilever.

structure, but several researchers have generated their codes [22, 23] or developed scripts [24] using these software's programming languages.

Structural optimization concerns on getting the required task of the mechanical system and maximizing its efficiency by an ordered procedure. At the beginning the design variables should be selected carefully. Then, limitations of these variables and system performance factors will be defined. By changing variable values, it is possible to see the change in these factors so we are able to determine the best combination among the design space. As design variables, the size of the members or mechanical properties of materials could be selected similar to size optimization, and the configuration of members is also another possible parameter as in shape optimization. Material distribution and layout are the parameter that is concerned in topology optimization. As the objective function, the most used one is cost function (related to total weight) to be minimized. Stress and buckling conditions are mostly used constraints in literature [18]. The aim is to optimize parts or units for specific load cases and extreme situations.

Figure 5 shows a sample application of topology optimization in finding the best material distribution. Minimizing objective function is acquired by checking different structure forms step by step. Each time design is narrowed down by selecting the best form among feasible sets.

3. Classification of methodologies

Topology optimization methods are mathematical techniques/approaches, and they can be programmed using different algorithms. These algorithms could be classified as follows: the criterion algorithm, the mathematical programming algorithm, and the intelligent algorithm.

The criterion algorithm obtains the optimality condition by the perceptual knowledge or the rational derivation. Result geometry will be gained by checking constraint violations and objective function value in an iterative way.

The perceptual criterion is usually the extension of the optimality condition of the full stress criterion of the size optimization. The rational criterion is derived usually by the Lagrange multiplier method of equality constraint. The ESO method is the typical criterion method.

Common mathematical programming algorithms like linear programming (LP) and nonlinear programming methods are also used in topology optimization of structures. The first attempts begin with using LP and successive LP methods later continued with sequential quadratic programming methods. Similar too criterion algorithm, mathematical programming algorithms are solved iteratively. Both stability and sensitivity of the structure are checked in each iteration. Of course it means that more calculation should be done for large-scale systems, and consequently low performance is observed for these cases. Fleury discussed the relationship between the criteria method and the mathematical programming method of size optimization. Fleury found that they both have given approximate results. This study refers still to the basics of the topology optimization [25, 26].

Genetic algorithm, simulated annealing algorithms, and particle swarm are the frequently used algorithms for topology optimization as the intelligent algorithm. The advantage of these algorithms is to keep it from too much calculations. The main idea is to search the optimum topology by checking only the objective function and constraints without calculating any gradients. On the contrary, solution speed can be slow, especially for large-scale system; finding optimum could take longer times [27, 28]. Several algorithms are also developed to combine topology optimization with additive manufacturing [29].

Two classes of approaches, the so-called material or micro-approaches and the geometrical or macro-approaches, are available [30, 31]. For the areas such as MEMS or biomaterial applications, classical continuum mechanics theories sometimes could not give accurate results. So, there are essential conceptual differences between these two types of approaches because of size effect.

Furthermore, another most commonly used classification merit of methodologies is if its discrete elements are used or not. The mainly used methods using discrete elements can be regarded, such as [18] ground structure approach (GSA) [21, 32], solid isotropic material with penalization (SIMP) method [33], homogenization method (HM) [34], evolutionary structural optimization (ESO) [35], and level-set method (LSM) [35]. On the other hand, the mainly used meshless methods are element-free Galerkin (EFG) [36], moving particle [37], and peridynamics [38]. Here, some of the studies post 2010 using these methodologies and their hybrids will be given under different headings.

3.1 Ground structure approach

Sokol and Rozvany [39] applied a hybrid method of linear programming and GSA to multi-load truss systems. Zhang et al. [40] combined GSA with simulated annealing to apply truss systems. Xu et al. [41] combined GSA with mixed integer linear programming for topology optimization of tensegrity structures. Zhang et al. [42] compared two different ground structure approach (macroelement and macropatch) on a skyscraper and arch bridge. Chun et al. [43] used a discrete filtering scheme in which thin bars are eliminated during reliability-based topology optimization. Gao et al. [44] considered principal stress trajectories to find the suitable nodal points to decrease the computational cost in building ground structure. Ha and Guest [45] applied the method to find the optimum 3D woven material structure and, in a later study, with their colleagues tested this structure [46]. Kosaka et al. [47] applied hybrid method of GSA and ESO to frame structures. Ramos and Paulino [48] considered the materials' nonlinear behavior to solve several topology optimization benchmarking problems. Shakya et al. [49] combined particle swarm optimization (PSO) algorithm with GSA in order to detect and remove useless elements of truss systems. Sokol [50] used GSA in the optimization of large-scale pin-jointed frames considering a new member adding strategy. Wang and Zhang [51] proposed a new approach, parallel optimization tactic, in topology optimization of multi-material compliant mechanism. Zegard and Paulino provided a code for 2D [52] and 3D [53] domains to prevent creating members not intersecting with others. Zhang et al. [54] worked on arranging optimum structure of multi-material composite material using Zhang-Paulino-Ramos design variable update scheme with Karush-Kuhn-Tucker conditions. Zhang et al. [21] used a different filtering scheme for the optimization of multi-materials (hyperelastic Ogden-based and bilinear materials).

3.2 Solid isotropic material with penalization method

Shao [55] has combined BESO with SIMP considering 3D printing applications. Lógó [56] has solved a continuum-type topology optimization problem considering uncertainties in load positions. Garcia-Lopez et al. [57] combined simulated annealing with SIMP to eliminate gray areas resulted by SIMP. Gebremedhen et al. [58] used SIMP to solve 3D stress-constrained topology optimization problems. Jantos et al. [59] used a new approach based on thermodynamics material modeling and not containing any filter and compared the results with SIMPs'. Jiao et al. [60] combined ESO with SIMP and used strain energy in their filtering function as sensitivity number. Kandemir et al. [61] proposed a new approach to define intermediate densities (gray areas) with new penalization factor. Marck et al. [62] applied SIMP to solve a multiobjective conductivity problem while using finite volume method (FVM) to solve the energy equation. Ospald and Herzog [63] used projected gradient method with SIMP to solve the structure problem of mold where short-fiber-reinforced polymer material is used in injection molding. Qiao et al. [64] applied the hybrid method of SIMP and BESO to a MBB beam and a cantilever beam and compared the results with literature. Schlinquer et al. [65] applied SIMP to design a mechanism used to amplify the displacement of a piezoelectric actuators. Tsai and Cheng [66] employed SIMP to design flywheel rotor having maximum stiffness. Wang et al. [67] combined topology and size optimization for a folding wing structural design. Yang et al. [68] accomplished topology optimization of an electric vehicle body by SIMP. Yang et al. [69] used SIMP for topology optimization of a hard disk drive. Yunfei et al. [70] applied SIMP to design a robot's upper arm. Zhang and Ren [71] proposed a new optimality criterion method concerning minimum compliance. Zhang et al. [72] presented a new approach to control the length scale of structural members. Zhang et al. [73] presented a method for cellular structures with multiple types of microstructures. Zuo and Saitou [74] introduced power functions with scaling and translation coefficients and the cost properties for multiple materials.

3.3 Homogenization method

Allaire et al. applied HM to structures made of periodically perforated material in 2D [75] and 3D [76]. Zhang and Khandelwal [77] presented a nonlinear homogenization method to be able to design metamaterials. Lee et al. [78] proposed asymptotic homogenization method to solve topology optimization problem of magnetic composite materials. El-Kahlout and Kiziltas [79] used together MATLAB code to integrate material model derived using homogenization theory with COMSOL and solved several design problems where periodic dielectric materials with desired properties are aimed. Noguchi et al. [80] proposed a level-set-based topology optimization method for the design of hyperbolic acoustic metamaterials using a high-frequency homogenization method. Larsen et al. [81] proposed a new approach based on HM extracting discrete structure from the continuum model. Milani and Bruggi [82] used an adaptive meshing algorithm with HM to optimize multistory masonry wall loaded up to failure. Groen and Sigmund [83] presented a projection method to get better meshes during topology optimization. Xia and Breitkopf [84] implemented a MATLAB code which uses energy-based homogenization approach rather than the asymptotic approach. Bruggi and Milani [85] arranged strut-and-tie models in reinforced concrete structures. Kaminakis et al. [86] used hybrid algorithm based on evolutionary algorithms in the design of microstructures having auxetic behavior.

3.4 Evolutionary structural optimization

Martínez-Frutos and Herrero-Pérez [87] used evolutionary algorithm to increase the efficiency of GPU and enable to solve with smaller amount of device memory. Daróczy and Jármai [88] proposed a new bidirectional evolutionary structural optimization (BESO) algorithm based on fluid dynamics analogy. Tomšič and Duhovnik [89] discussed simultaneous topology and size optimization of trusses. Abdi et al. [90] used a combination of ESO with XFEM which uses isoline design approach. Ansola et al. [91] used ESO to optimize compliant mechanism under concentrated and thermal loads. Aulig and Olhofer [92] combined a neuro-evolution algorithm with a gradient-based optimizer and later proposed another algorithm considering state-based representation [93]. Azamirad and Arezoo [94] combined programming environment with Abaqus FEM software to optimize die components. Bureerat and Sriworamas [95] proposed multiobjective real-code population-based incremental learning (RPBIL) and a hybrid algorithm of RPBIL with differential evolution (DE) (termed RPBIL-DE) to solve water distribution network. Chen et al. [96] used ESO to optimize the rotary lobe of root vacuum pumps. Chen [97] used modified ESO algorithm for the optimization of plate structure under harmonic loading. Cho et al. [98] obtained optimum topology for the inner reinforcement of a vehicle's hood having uncertainties in variables. Finotto et al. [99] used an algorithm combination of ground structure approach, nonlinear finite element analysis, and quantum-inspired evolutionary algorithms. Garcia-Lopez et al. [100] used multiobjective evolutionary algorithm handling uncertainties and also giving the Pareto frontier solutions to let user select the best solution. Greiner and Hajela [101] used multiobjective evolutionary algorithm using reunification criterion to increase search efficiency. Huang and Xie [102] used BESO utilizing an alternative material interpolation scheme. Huang et al. [103] used BESO to optimize the topology of PBC made of two-phase composites. Zuo and Xie [104] used ESO letting limiting displacement. Jantos et al. [105] added a control mechanism for growth factor where at each step Lagrange multiplier is used to find optimum. Jia et al. [106] used hybrid of ESO with LSM. Kaminakis et al. [107]

proposed hybrid method of Particle Swarm Optimization and differential evolution in the design of microstructures. Kunakote and Bureerat [108] compared Pareto archive evolution strategy (PAES), population-based incremental learning (PBIL), non-dominated sorting genetic algorithm (NSGA), strength Pareto evolutionary algorithm (SPEA), and multiobjective particle swarm optimization (MPSO). Li et al. [109] used a combination of SIMP and ESO. Li et al. [110] used BESO method in the design of hinge-free compliant mechanisms. Maleki Jebeli and Shariat Panahi [111] used GA as evolutionary algorithm to optimize the material property distribution in FG structures. Okamoto et al. [112] enhanced genetic algorithm, immune algorithm, additional search in the restricted design space with enabling island, and void distribution during FEM analysis to solve a typical magnetic circuit problem. Picelli et al. [113] used BESO to free vibration problems of acoustic-structure systems. Riehl and Steinmann [114] employed the traction method to define descent directions for shape variation. Shi et al. [115] used APDL and UIDL to implement BESO in ANSYS to improve results. Sun et al. [116] applied BESO a cantilever composite laminate under uniform in-plane pressure. Tominaga et al. [117] used GA algorithms for magnetostatic shielding to minimize the magnetic flux intensity in a specified region. Wang et al. [118] used to optimize constrained damping layer structure. Fritzen et al. [119] taken nonlinear elastoviscoplastic microscopic RVE into account at all points of the macroscopic design domain by using BESO. Later, Xia et al. [120] introduced a damping scheme on sensitivity numbers to the same approach. Zhu et al. [121] used bidirectional evolutionary level-set method allowing automatic hole generation. Zuo et al. [27] enhanced the BESO method to multiple constraints of displacement and frequency in addition to the amount of material usage.

3.5 Level-set method (LSM)

Allaire et al. [122] applied LSM with enabling local mesh modifications. Chen and Chen [123] considered geometric uncertainty and related problems. Van Dijk et al. [124] used uses a direct steepest-descent update of the design variables in a LSM. Dunning and Alicia Kim [125] developed a third dimension for 2D problems to adjust new hole positions and to prevent violations with boundaries. Emmendoerfer and Fancello [126] minimized mass under stress constraints using an augmented Lagrangian approach. Gomes et al. [127] interested in the reduction of the design space dimension by the help of a GUI. Guo et al. [128] used LSM in stress-related topology optimization problems. Otomori et al. applied LSM to the design of electromagnetic cloaks using a ferrite material [129] and a light-scattering layer for solar cell applications [130]. Guo et al. [131] developed a local and explicit feature control scheme. James et al. [132] used isoparametric finite element, and James and Martins [133] used a body-fitted, nonuniform finite element mesh to overcome irregular shape problems. Jang et al. [134] considered geometric uncertainties in the production of microsystems. Lim et al. [135] applied to magnetic actuator design problems. Liu et al. [136] adopted extended finite element method (XFEM) with unified structural optimization model help to cover the topology, shape, and sizing optimization at the same time. Luo et al. [137] combined meshless Galerkin method with LSM. Makhija and Maute [138] applied a generalized Heaviside enrichment strategy with XFEM formulation. Mohamadian and Shojaee [139] combined binary level-set method and Merriman-Bence-Osher scheme. Otomori et al. [140] used LSM in the design of negative permeability dielectric metamaterials. Shojaee and Mohammadian [141] combined piecewise constant level-set (PCLS) method with a MBO scheme. Shu et al. [142] used LSM to minimize frequency response which results in the reduction in the vibration of structure.

Shu et al. [143] used LSM in the design of coupled structural-acoustic system with a focus on interior noise reduction. Suresh and Takalloozadeh [144] used LSM considering stress constraints. Xia et al. [145] used LSM to maximize the simple or repeated first eigenvalue of structure vibration. Xia et al. [146] built a strict 0–1 model considering stress to be minimized. Xia et al. [147] optimized both structure and support using traction free and Dirichlet boundaries separately. Yamasaki et al. [148] proposed a method combined application of boundary element mesh with LSM. Zhu and Zhang [149] used LSM without re-initialization for the optimization of compliant mechanisms. Zhu et al. [150] combined projection Lagrangian method with piecewise constant level-set functions to manage the optimization for elliptic boundary value problems. Zhu et al. [151] used LSM to optimize hinge-free compliant mechanisms with multiple outputs. Zhu and Zhang [152] developed an accelerated level-set evolution algorithm by adding an extra energy function to be able to optimize the distributed compliant mechanisms. Zhu et al. [153] developed a new LSM to manage multiobjective optimization of hinge-free compliant mechanisms.

3.6 Meshless methods

Lin et al. [154] generated a method mimicking leaf venation and using element-free Galerkin method to design heat conduction channels. Wang and Luo [155] proposed a meshless Galerkin level-set method using compactly supported radial basis functions to construct the meshless shape functions. Cui et al. [156] proposed a new method based on SIMP and using EFG method for multi-material optimization problems. Zhao [157] developed a new approach based on Pareto frontier solutions using EFG method. He et al. [158] combined density variable approach with EFG to optimize geometrically nonlinear structures. Evgrafov [159] proposed a method based on SIMP combined with Petrov-Galerkin methods based on minimizing the squared residual. Khan et al. [160] used EFG with LSM and also implemented sensitivity analysis. Gong et al. [161] developed a new method, particle moving, based on EFG considering density gradient and combined it with SIMP. Hur et al. [162] used a Spline-based meshfree method where nonuniform rational B-spline functions are used to smooth trimmed boundaries. Ren et al. [163] used a method combination of EFG and SIMP to design a two-material micro-compliant mechanism under stress constraints. Zhang et al. [164] applied a combined method of SIMP and direct coupling method of FE and EFG methods to decrease computational cost of meshless methods. Ai and Gao [165] integrated a parametric level-set method with a meshless method based on compactly supported radial basis functions. Wang et al. [166] applied EFG to the design of large displacement compliant mechanisms having geometrical nonlinearity. Yang et al. [167] applied EFG to the design of continuum structures under displacement constraints. Kefal et al. [38] combined BESO with a new meshless method peridynamics. Zheng et al. [168] used a combination of SIMP and EFG to optimize free vibrating continuum structures. Zhang et al. [169] used a directly coupled FE and EFG to optimize nonlinear hyperelastic structures. Luo et al. [36] used dual-level point-wise density approximation with EFG. Wu et al. [170] improved EFG by adding moving least squares approximation. Zheng et al. [171] used EFG to optimize geometrically nonlinear continuum structures. Zhao [172] combined BESO with EFG.

4. Emerging areas and recommendations

Sigmund and Maute [11] drawn a good framework on the classification of methodologies, and they pointed an important spot that differences between topology optimization approaches become small and an approach evolves into the other by the time such as evolutionary methods are converging towards discrete SIMP schemes. However, this trend has gone forward using hybrid approaches rather than becoming similar techniques to keep all the approaches having their advantages and limitations. There are many studies using hybrid methodologies given before under different headings, but there is still room for new applications. Especially from evolutionary algorithms perspective, using new optimization algorithms will enable to improve methodologies advanced up to now.

Another important area to work on is how uncertainties are handled. Topology optimization of small sized systems brings researchers to the position where small changes should be taken into account as todays' technology is covering nano-sized systems beyond MEMS. In any case when changes are formed either because of manufacturing errors or that applied loads has caused comparatively large deformations on members, it will not be possible to use precise geometry and crisp size values in the optimization stage. So, handling uncertainties such as using fuzzy systems is still an open field to study.

Lastly, another rapidly growing area at the last decade is rapid prototyping. Even though there are abundant studies in literature (over a hundred studies could be easily found [173]), new algorithms on the application of BESO, handling composite/functionally graded materials, and considering support and structure in the meantime are the promising areas to study.

In addition to the aforementioned emerging areas, researchers are encouraged to study (1) to develop the efficiency of standard methods; (2) to construct new benchmarking problems; (3) to consider several constraints rather than buckling, stress, or displacement of such natural frequency; (4) to adapt meshes to nonlinear geometries with a more accurate way; (5) to develop GUIs to help researcher to observe/interfere the optimization stage; and (6) to implement new meshless methods rather than EFG such as peridynamics.

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